1. Does there exist irrational numbers u and v such that u^v is rational?
   Solution: Yes. Consider the number $x = \sqrt{2}^{\sqrt{2}}$.

   - If x is rational, we are done by picking $u = v = \sqrt{2}$.
   - Else, let $u = x = \sqrt{2}^{\sqrt{2}}$ and $v = \sqrt{2}$. We now have $u^v = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \left(\sqrt{2}\right)^2 = 2$ and now we are done.