1. We build a computer, where the real numbers are represented using 5 digits as explained below:

S	A	B	C	E

where

- S is the sign bit; 0 is positive and 1 is negative
- A, B, C: First three significant digits in decimal expansion with decimal point occuring between A and B
- E is the exponent in base 10 with abias of 5
- All digits after the third significant digit are chopped off
- $+0$ is represented by setting $S = 0$ and $A = 0$; B, C, E can be anything
- -0 is represented by setting $S = 1$ and $A = 0$; B, C, E can be anything
- $+\infty$ is represented by setting $S = 0$ and $A = B = C = E = 9$
- $-\infty$ is represented by setting $S = 1$ and $A = B = C = E = 9$
- Not A Number is represented by setting S to be other than 0 and 1.

For example, the number $\pi = 3.14159\ldots$ is represetned as follows. Chopping off after the third significant digit, we have $\pi = +3.14 \times 10^0$. Hence, the representation of π on our machine is:

0	3	1	4	5

The number $-0.001259\ldots$ is represetned as follows. Chopping off after the third significant digit, we have -1.25×10^{-3}. Hence, the representation of on our machine is:

1	1	2	5	2

Now answer the following questions:

(a) How many non-zero Floating Point numbers (from now on abbreviated as FPN) can be represented by our machine?

(b) How many FPN are in the following intervals?
   - $(9, 10)$
   - $(10, 11)$
   - $(0, 1)$

(c) Identify the smallest positive and largest positive FPN on the machine

(d) Identofy the machine precision

(e) What is the smallest positive integer not representable exactly on this machine?

(f) Consider solving the following recurrence on our machine:

$$a_{n+1} = 5a_n - 4a_{n-1}$$

with $a_1 = a_2 = 2.932$. Compute a_n for $n \in \{3, 4, 5, 6, 7\}$ on our machine (work out what the machine would do by hand). Note a_1, a_2 would be chopped to three significant digits to begin with. Next note that at each step in the recurrence $5a_n$ and $4a_{n-1}$ would be chopped down to the first three significant digits before the subtraction is performed.

2. Consider the following integral:

$$I_n = \int_0^1 x^{2n} \sin(\pi x) dx$$

(a) Obtain a recurrence for I_n in terms of I_{n-1}. (HINT: Integration by parts)

(b) Evaluate I_0 by hand

(c) Use the recurrence to obtain I_n for $n \in \{1, 2, \ldots, 15\}$ in MATLAB

(d) Use wolframalpha to obtain I_n by directly performing the integeral for $n \in \{1, 2, \ldots, 15\}$.

(e) Explain your observation